

tax  $\tau_F w^{***}$  that the firm has to pay to the government, and the total wage cost  $(1 + \tau_F) w^{***}$  of the firm.

## 5.4 Equivalence

Finally, consider a situation where the government chooses the tax rate  $\tau_F$  in a way that the wage that the worker receives in equilibrium,  $w^{***}$ , is exactly the same as the after-tax wage that the worker would receive in an economy with taxation only on the worker side,  $(1 - \tau_w) w^{**}$ .

Replot the graph from question 5.2. Now, into the same graph, plot the impact of the tax rate  $\tau_F$  imposed on the firm that is precisely high enough to give the worker the same equilibrium after-tax wage as in the case when the government uses tax  $\tau_w$  imposed on the worker.

Notice that to achieve that, it must be true that  $w^{***} = (1 - \tau_w) w^{**}$ . Argue that in this case, it must also be true that the total wage cost of the firm must be the same under both taxation schemes — in other words, it must be true that the total wage cost of the firm when the government uses the tax imposed on the firm,  $(1 + \tau_F) w^{***}$ , is the same as the total wage cost of the firm when the government uses the tax on the worker,  $w^{**}$ .

Imagine that the government wants to switch from a given tax rate  $\tau_w$  imposed on the worker to a tax rate  $\tau_F$  imposed on the firm. Using the two above conditions, derive the tax on the firm that allows this switch while not changing worker's after-tax wages or firm's total wage cost.

## 5.5 Equivalent budget

Show that when the tax on the firm  $\tau_F$  and tax on the worker  $\tau_w$  are chosen in a way that satisfies the conditions in question 5.4, then, in both cases, the total tax revenue in the government is the same.

## 6 A model of incarceration (not for the exam)

**Background:** *While the question of who is sent to jail and for how long does not seem to be much of an economic topic, the opposite is actually true. For instance, the U.S. has by far the highest rate of incarceration (fraction of adult people in jail) of all developed countries (almost ten times*

higher than Germany). Among some population groups, like young black males, the incarceration rate is so high that it has a severe economic impact (for instance, on inequality in society), and is subject to extensive research.

We will consider an extremely simplified model of incarceration. It omits a lot of detail but in all your answers, you should stick to the model, and not make up justifications that are outside of the model framework.

Assume that the population size is fixed at  $L$ . Denote  $J_t$  the number of people currently (in period  $t$ ) in jail, and  $N_t$  the number of people who are not in jail. Assume that in every period, a fraction  $r$  of the people in jail is released from jail, while a fraction  $s$  of people who are **not** in jail is sentenced and sent to jail.

## 6.1 Dynamics

Write down two equations that described the dynamics of the model. The first equation is a ‘resource constraint’ on the total population size in every period  $t$ . The second equation captures the change in the number of people in jail between periods  $t$  and  $t + 1$  as the difference between the ‘inflow’ into jail, and ‘outflow’ out of jail.

## 6.2 Steady state

What condition needs to be satisfied in the steady state of this model?

## 6.3 Steady state rate

Find the **steady state** rate of incarceration  $j^*$ , defined as the steady state ratio of jail population over total population.

## 6.4 Release

Imagine that the ‘economy’ in this model is in steady state in year 2024. On December 31, 2024, there is a presidential amnesty and 10% of people are released from jail in addition to the people who would be released normally during 2024. There is no other change in the parameters.

**In year 2025**, the number of people who are sentenced and sent to jail will be **higher, the same, or lower** than the number of people released from prison (in the same year)?

## 6.5 Sentences

Imagine that the courts decide they should be tougher. In particular, they decide that the right thing to do is to give longer sentences (i.e., each sentenced person spends a longer period in jail), but they **do not** sentence more people (i.e., they do not sentence people who are currently out of jail at a higher rate).

How would you model this change in policy in the context of the current model? I.e., which parameter of the model should change and how? What will happen to the steady state rate of incarceration  $j^*$ ?